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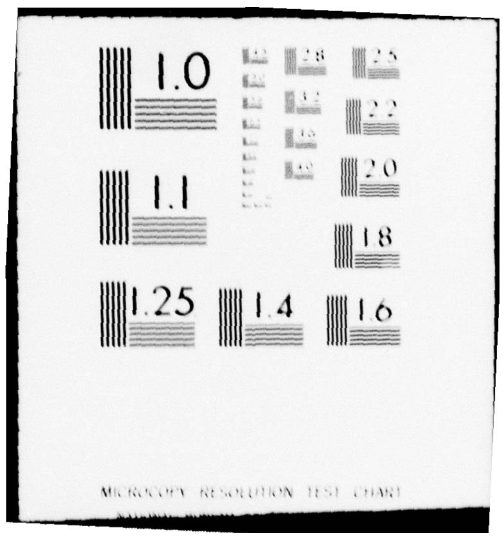
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self-adjoint; asymptotically stable; orthogonal sequence; conjugate points; Liapunov functions; domain of attraction; conjugacy function; minimax function; characteristic numbers; characteristic functions.

x(double dot)
beta(x(dot))

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Appreciably larger domains of attraction were found for asymptotically stable equilibrium points of equations of the form $\ddot{x} + \beta(\dot{x}) + f(x) = 0$; a generalization of a classical result for orthogonal polynomials was obtained; a new comparison theorem for conjugate points of equations $(r(x)y')' + p(x)y = 0$ is given; the last yields a comparison theorem for the minimax function.

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To study an asymptotically stable equilibrium point at $x = 0$ of a differential equation

$$\ddot{x} + \beta\dot{x} + f(x) = 0 \quad (\beta > 0)$$

one first writes

$$(1) \qquad \begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\beta y - f(x). \end{aligned}$$

Under reasonable restrictions on $f(x)$, the "standard" [cf. 1] Liapunov function

$$V(x, y) = y^2 + 2\int_0^x f(x)dx$$

yields a domain of attraction of the origin for (1).

In [2] it is shown that the Liapunov function

$$(2) \qquad V(x, y) = (k - x)\frac{y^2}{2} + \int_0^x (k - x)f(x)dx,$$

with a suitable choice of the constant $\underline{k}$ yields an appreciably larger domain of attraction. It is then shown that a still larger domain may be obtained using the Liapunov function

$$V(x, y) = e^{-\alpha x}\frac{y^2}{2} + \int_0^x e^{-\alpha x}f(x)dx,$$

for a suitable choice of the constant $\underline{\alpha}$.

Next consider the differential system

$$[r(x)y']' + \lambda p(x)y = 0,$$
$$\alpha y(a) + \beta y'(a) = 0,$$
$$\gamma y(b) + \delta y'(b) = 0,$$

where $r(x)$, $p(x)$ are continuous and positive on $[a, b]$, and $\alpha^2 + \beta^2 > 0$, $\gamma^2 + \delta^2 > 0$. Let $0 < \lambda_1 < \lambda_2 <\ldots \to +\infty$ be the characteristic numbers and $\{y_n(x)\}_1^\infty$ the corresponding characteristic functions of this system. It is well known that the functions $y_n(x)$ form an orthogonal sequence on $[a, b]$ with respect to the weight function $p(x)$.

In [3], it is shown that the functions $y_1'(x)$, $y_2'(x)$, ... form an orthogonal sequence on [a, b] with weight function $r(x)$.

Consider a pair of self-adjoint differential equations

$$[r(x)y']' + p(x)y = 0,$$

(3)

$$[r_1(x)y']' + p_1(x)z = 0,$$

where $p(x)$, $p_1(x)$ are continuous and $r_1(x)$ and $r(x)$ are positive and of class C" on an interval I of the $\underline{x}$-axis.

It is shown in [3] that if nonnull solutions $y(x)$ and $z(x)$ of (3) are such that

$$z(a) = z(b) = 0 \quad (a < b),$$

$$y(a) = 0,$$

and if

$$\left(\frac{r'}{r}\right)^2 - 2\frac{r''}{r} + 4\frac{p}{r} \geq \left(\frac{r_1'}{r_1}\right)^2 - 2\frac{r_1''}{r_1} + 4\frac{p}{r_1} \quad (a \leq x \leq b),$$

with strict inequality holding for at least one point of [a, b], then

$$y(c) = 0 \quad (a < c < b).$$

This comparison theorem leads to a comparison theorem for the zeros of the derivatives of a solution. Specifically, let $\delta(x)$ be the conjugacy function [cf. 4] for the differential equation

(4) $$[r(x)y']' + p(x)y = 0.$$

Here $r(x)$, $p(x)$ are continuous and positive on an interval I of the $\underline{x}$-axis. The conjugacy function $\delta(x)$, it will be recalled, is the (positive) distance from a point $x \in I$ to its first conjugate point. The minimax function $\delta_1(x)$ associated with (4) is the (positive) distance from a point $x \in I$ where the derivative of a solution of (4) vanishes to the derivative's next greater zero.

The result then is this. If

$$\left(\frac{r'}{r}\right)^2 + 3\left(\frac{p'}{p}\right)^2 \geq 2\left(\frac{p''}{p} + \frac{r''}{r}\right), \tag{5}$$

with strict inequality holding at at least one point,

$$\delta(x) < \delta_1(x).$$

When the inequality (5) is reversed,

$$\delta_1(x) < \delta(x).$$

Work was begun on two other projects, but there is nothing yet ready to report. One project has to do with the detection of singularities of holomorphic functions, and the other involves an attempt to determine new general methods of constructing Liapunov functions. The Principal Investigator hopes to re-examine these problems and others at some time in the future.

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